The Importance of Realistic Starting Models for Hydrodynamic Simulations of Stellar Collisions

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ABSTRACT

We demonstrate the necessity of using realistic stellar models taken from stellar evolution codes, as opposed to polytropes, for starting models in smoothed particle hydrodynamics calculations of collisions between main sequence stars. Evolved stars have mean molecular weight gradients, which affect their entropy profiles and therefore affect how they react during a collision. The structure of stellar collision products of polytrope parent stars is significantly different from that of collision products of realistic parent models. These differences strongly affect the future evolution of the collision products, particularly products of collisions between unequal mass stars which have undergone significant chemical evolution. The use of polytropes as parent star models is likely to result in qualitatively mistaken results for the structure of the collision product.

Subject headings: hydrodynamics – stars: evolution

1. Introduction

Hydrodynamical codes provide a useful numerical tool for determining what happens during a collision between two main sequence stars. In particular, the smoothed particle hydrodynamic (SPH) technique has been used extensively in the past decade to investigate the possibility that blue stragglers can be created by stellar collisions in globular clusters (Benz & Hills 1987, Goodmann & Hernquist 1991, Lombardi, Rasio & Shapiro 1996, Sandquist, Bolte & Hernquist 1997). The emphasis of these groups has been to determine the internal structure of stellar collision products. These models can then be compared to blue stragglers in globular clusters, to help determine the creation mechanisms for blue stragglers, or to give information about the dynamics of the cluster as a whole.

When running an SPH simulation of a stellar collision, one needs to specify the entropy (or pressure and density) profiles of the parent stars. Most groups used polytropes with polytropic indices of $n = 1.5$ (for convective stars) and $n = 3$ (for radiative stars) as approximations to the stars. While these polytropes are reasonable approximations to zero age main sequence stars, they are not sufficient representations of evolved stars. The most important difference between the two kinds of stellar models is that evolved stars have gradients of mean molecular weight. In this paper, we deal specifically with main sequence parent stars, and not with white dwarfs which are well represented by $n = 3$ polytropes.

An SPH code takes the two parent stars and divides them up into many fluid elements. For head on collisions, the code creates a collision product by, in essence, sorting the fluid in a configuration with the quantity $A = P/\rho^{\Gamma_1}$ increasing outward (the dynamical stability criterion), where P is pressure, ρ is density and the adiabatic exponent $\Gamma_1 = 5/3$ for an ideal gas. While real stars are not exactly ideal gases, the low mass stars we are considering here are dominated by gas pressure, with radiation pressure contributing at most 5% to the total pressure. The quantity A is closely related to entropy, since in the absence of shocks

both are conserved by fluid elements during a collision. For convenience, we will refer to A itself as the entropy. The entropy of a fluid element can be increased by shock heating, but since collisions in globular clusters are relatively gentle, this effect is not dominant. Therefore, sorting the two parent stars by entropy yields a reasonable guess for the product of a given head on collision.

During the main sequence lifetime of a star, its central temperature remains almost constant, since nuclear burning is a self-regulating process. The central pressure of a star is also approximately constant, since this is the pressure which is necessary to support a star of that mass. Therefore, for a star with an ideal gas equation of state $P = \frac{R\rho T}{\mu}$, the ratio of density to mean molecular weight must also remain constant. As the mean molecular weight increases, the density must also increase in order to keep the pressure and temperature constant. Since the density is in the denominator of the entropy, and since it is raised to a power greater than one, any increase in density results in a significant decrease in entropy. An evolved star has burnt much of its hydrogen to helium, which increases the mean molecular weight of the central material of the star. The central entropy of this evolved star will be significantly less than that of a corresponding unevolved star, or a polytrope.

2. A Demonstration

Take, for example, a $0.8M_{\odot}$ star at an age of 15 Gyr, a turnoff star in a typical globular cluster. It has burnt almost all of its central hydrogen to helium, increasing the mean molecular weight at the center by a factor of about 2.5. Since the mean molecular weight at the center has increased, the central entropy has decreased (see Figure 1). The realistic stellar models have been calculated using the Yale Rotating Evolution Code (YREC, Guenther *et al.* 1992). The polytrope used to model an unevolved $0.8M_{\odot}$ star has

a polytropic index of $n = 3$.

Also consider a $0.4 M_\odot$ star at an age of 15 Gyr. This star has completed only a small fraction of its main sequence lifetime, and therefore has not changed its mean molecular weight profile significantly. The evolved entropy profile looks quite similar to the entropy profile of the $n = 1.5$ polytrope approximation of an unevolved star of the same mass (see Figure 1). The decrease in entropy near the surface is a result of approximating the equation of state as that of a fully ionized ideal gas.

From figure 1, we can ascertain the general properties of a product that would result from the collision between any two of these stars. The portions of the two stars with the lowest entropy would settle to the center of the collision product, while that with the highest entropy would end at the surface. Therefore, if we use the two polytrope models as our parent stars, the entire $0.4 M_\odot$ star will become the new core of the collision product, displacing the $0.8 M_\odot$ star outward. The collision product would then have a core of unburnt hydrogen from the $0.4 M_\odot$ star, surrounded by a layer of helium-rich material from the core of the $0.8 M_\odot$ star. However, a significantly different outcome occurs if we use the evolved parent stars. In this case, the center of the $0.8 M_\odot$ star has the lowest entropy and consequentially sinks to the center of the collision product. The core of the $0.4 M_\odot$ star has a much higher entropy, and ends up further out in the collision product. This star will have a core which is depleted of hydrogen from the core of the $0.8 M_\odot$ star, and an outer layer which is almost completely unburnt.

We can also consider a collision between two $0.8 M_\odot$ stars. We expect in this case that there will be no significant differences in the fluid distribution between using polytropes and using realistic parent stars. Although the values of the entropy curves are different, the overall shape is generally the same: the lowest entropy occurs in the center of the star, and the entropy increases monotonically to the surface. Therefore, when two of these stars

collide, the centers of the two stars will fall to the center of the product, and the outer layers will stay on the outside.

We have performed new simulations of a head-on collision between a $0.8M_{\odot}$ and a $0.4M_{\odot}$ star, and between two $0.8M_{\odot}$ stars using a modified version of the SPH code developed by Rasio (1991). The parent models were created using the density and pressure profiles from the realistic evolved models calculated by YREC. The $0.4M_{\odot}$ and $0.8M_{\odot}$ stars were represented by 7.5×10^3 and 1.5×10^4 equal mass SPH particles respectively. Details of the SPH scheme are presented in Lombardi *et al.* (1996). We compared the results of these calculations to the results of previous SPH calculations which used polytropes as parent stars (see cases A and G in Lombardi, Rasio & Shapiro 1996). The structures of the merger products are compared in figure 2 (for the $0.8M_{\odot} + 0.4M_{\odot}$ collision) and figure 3 ($0.8M_{\odot} + 0.8M_{\odot}$). The results are essentially as we predicted above. This can be seen best in the plots of helium fraction Y versus mass fraction, since the highest Y material was in the center of the $0.8M_{\odot}$ star at the beginning of the collision.

Using the technique outlined in Sills *et al.* 1997, we have used the results of these collisions as the starting models for stellar evolution calculations. The results are presented in figure 4. As expected, the evolutionary tracks for the two cases of the $0.8M_{\odot} + 0.4M_{\odot}$ collision are very different. The collision of two polytropes contains a significant amount of hydrogen in its core (from the $0.4M_{\odot}$ parent). Therefore, the evolutionary track of this collision product shows a significant main sequence of 1.4×10^9 years from point 1 to point 2. However, the product of the collision between realistic parent stars did not have much hydrogen in its core, and so its main sequence (point 1 to point 2) is almost non-existent, lasting only 3.4×10^6 years. The two tracks for the $0.8M_{\odot} + 0.8M_{\odot}$ collisions are essentially the same from the main sequence onward since the chemical composition and temperature profiles of the collision product were not significantly affected by the choice of parent star.

3. Conclusions

We advocate the use of realistic stellar models, taken from stellar evolution codes, for starting models in hydrodynamic calculations. In particular, polytropic models are insufficient to model main sequence and giant stars which have undergone significant chemical evolution. While previous groups have made reasonable guesses for the structure of turnoff mass stars in globular clusters, the fact that they did not use realistic parent models significantly affects the structure of the collision product. Therefore, any further evolution of these stars does not adequately represent the evolution of a real collision between globular cluster stars. The differences will be significant for work which draws conclusions about the structure of the collision product or which uses the results of hydrodynamic calculations as the starting models for stellar evolution calculations.

The general conclusions of Lombardi *et al.* 1996 and Sills *et al.* 1997 are not invalidated by this work. It is still true that significant mixing does not occur either during the collision or during the thermal relaxation phase after the collision. However, the lifetime of the star on the main sequence and its subsequent evolution is significantly different from previous work, especially in collisions between unequal mass parent stars.

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REFERENCES

Benz, W. & Hills, J. G. 1987, ApJ, 323, 614

Goodmann, J., & Hernquist, L. 1991, ApJ, 378, 637

Guenther, D. B., Demarque, P., Kim, Y.-C., & Pinsonneault, M. H., 1992 ApJ, 387, 372

Huang, K. 1987, Statistical Mechanics, (New York: John Wiley and Sons)

Lombardi, J. C., Rasio, F. A., & Shapiro, S.L. 1996, ApJ, 468, 797

Rasio, F. A. 1991, PhD Thesis, Cornell University

Sandquist, E., Bolte, M., & Hernquist, L. 1997, ApJ, 477, 335

Sills, A., Lombardi, J. C., Bailyn, C. D., Demarque, P., Rasio, F. A., & Shapiro, S. L. 1997 ApJ, in press

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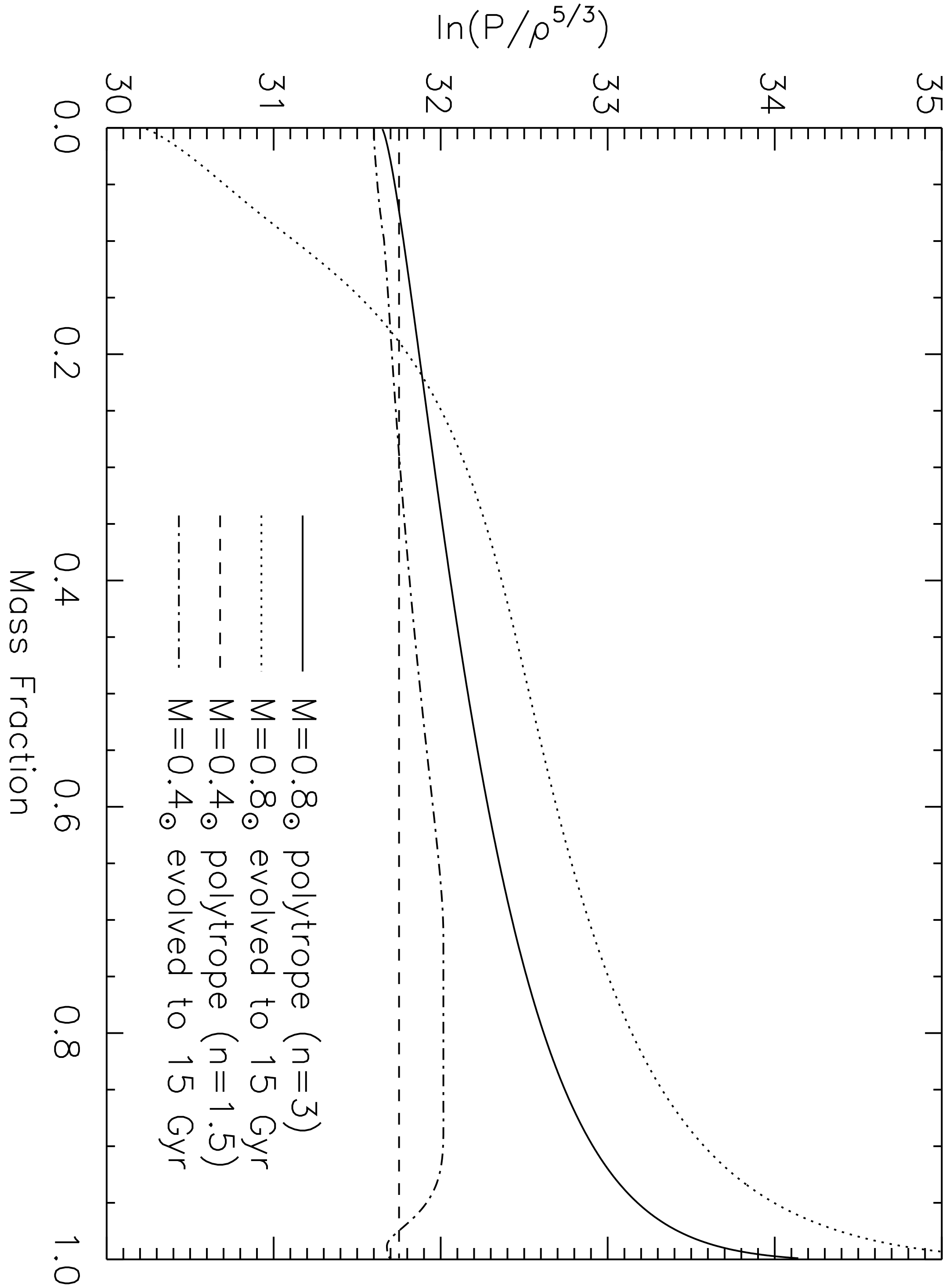
Fig. 1.— Entropy profiles of polytropes and realistic parent stars evolved to 15 Gyr. Two masses are shown here: $0.4M_{\odot}$ (dashed and dot-dashed lines) and $0.8M_{\odot}$ (solid and dotted lines). Note the difference in shape between the polytrope and realistic curves of the same mass. The pressure P and density ρ are evaluated in cgs units.

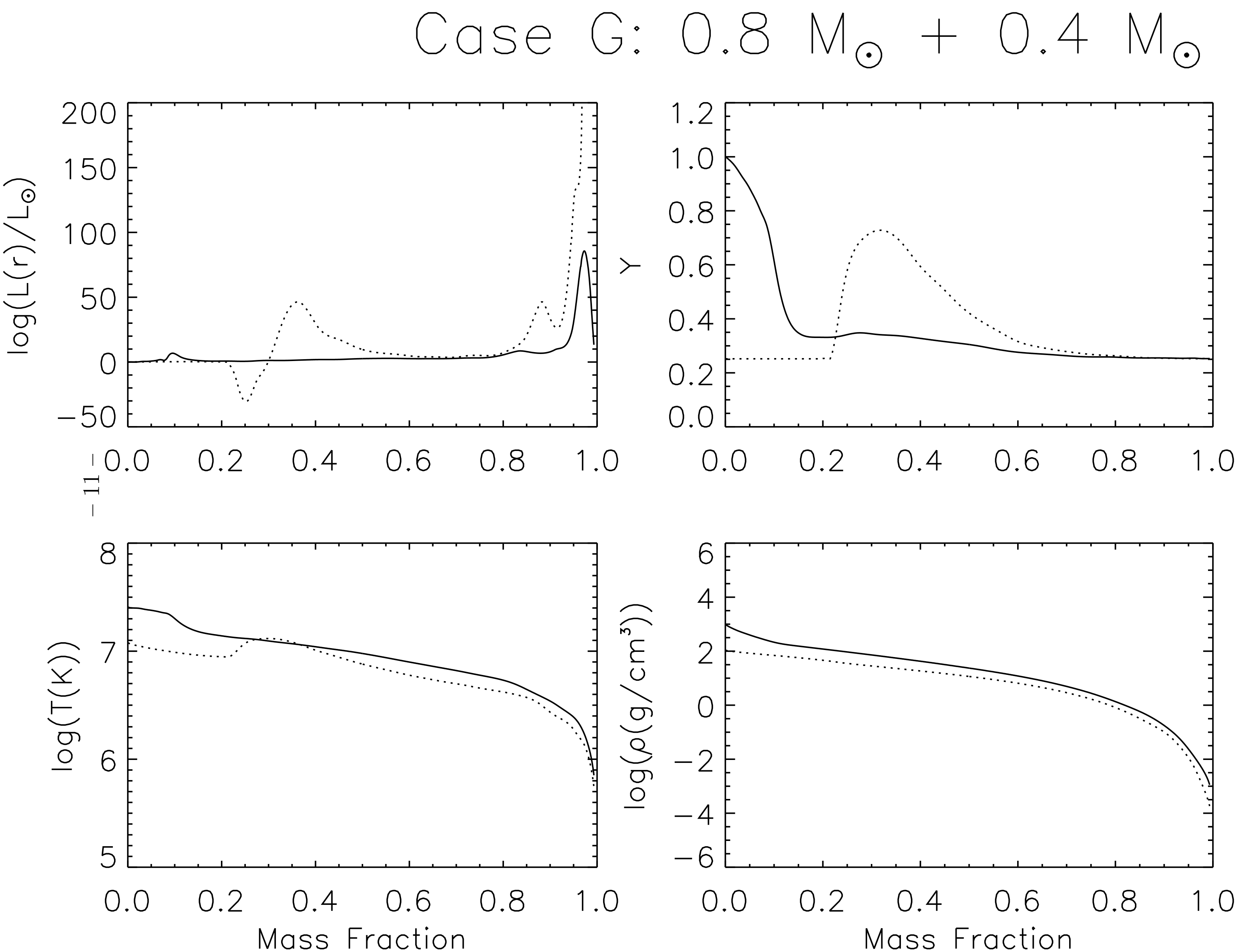
Fig. 2.— Comparison of the structure of the collision products for an $0.8M_{\odot} + 0.4M_{\odot}$ collision. Solid line is the product of two realistic parent stars, and the dotted line is the product of two polytrope models. Note especially the large difference in chemical profiles between the two cases.

Fig. 3.— Same as figure 2 for a $0.8M_{\odot} + 0.8M_{\odot}$ collision. Since these stars are the same mass, there are only subtle differences between the polytrope and the realistic parent star collisions.

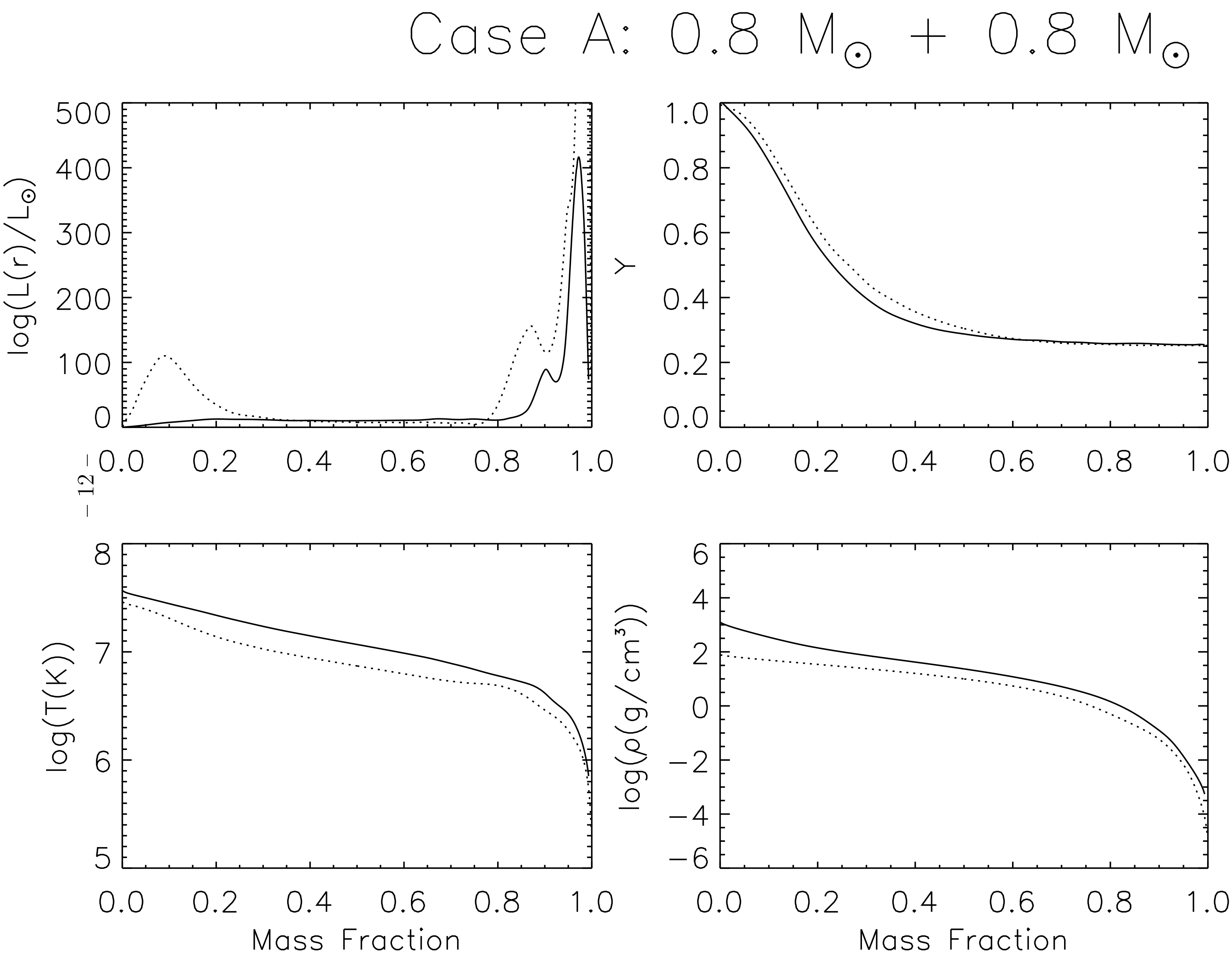
Fig. 4.— Evolutionary tracks of the merger products presented in figures 2 and 3. Solid lines are the tracks for collision products from realistic models, and dotted lines are from polytropes. The upper panel shows tracks for collisions between a $0.8M_{\odot}$ and a $0.4M_{\odot}$ star, while the lower panel shows tracks for a collision between two $0.8M_{\odot}$ stars. The labels show significant evolutionary points along the track. Point 0 is the position of the collision product immediately after the collision. Point 1 is the equivalent of the zero age main sequence, and point 2 is the terminal age main sequence, when the star has no more hydrogen in its core. Point 3 is the base of the giant branch. The main sequence lifetimes (between points 1 and 2) of each star are given.

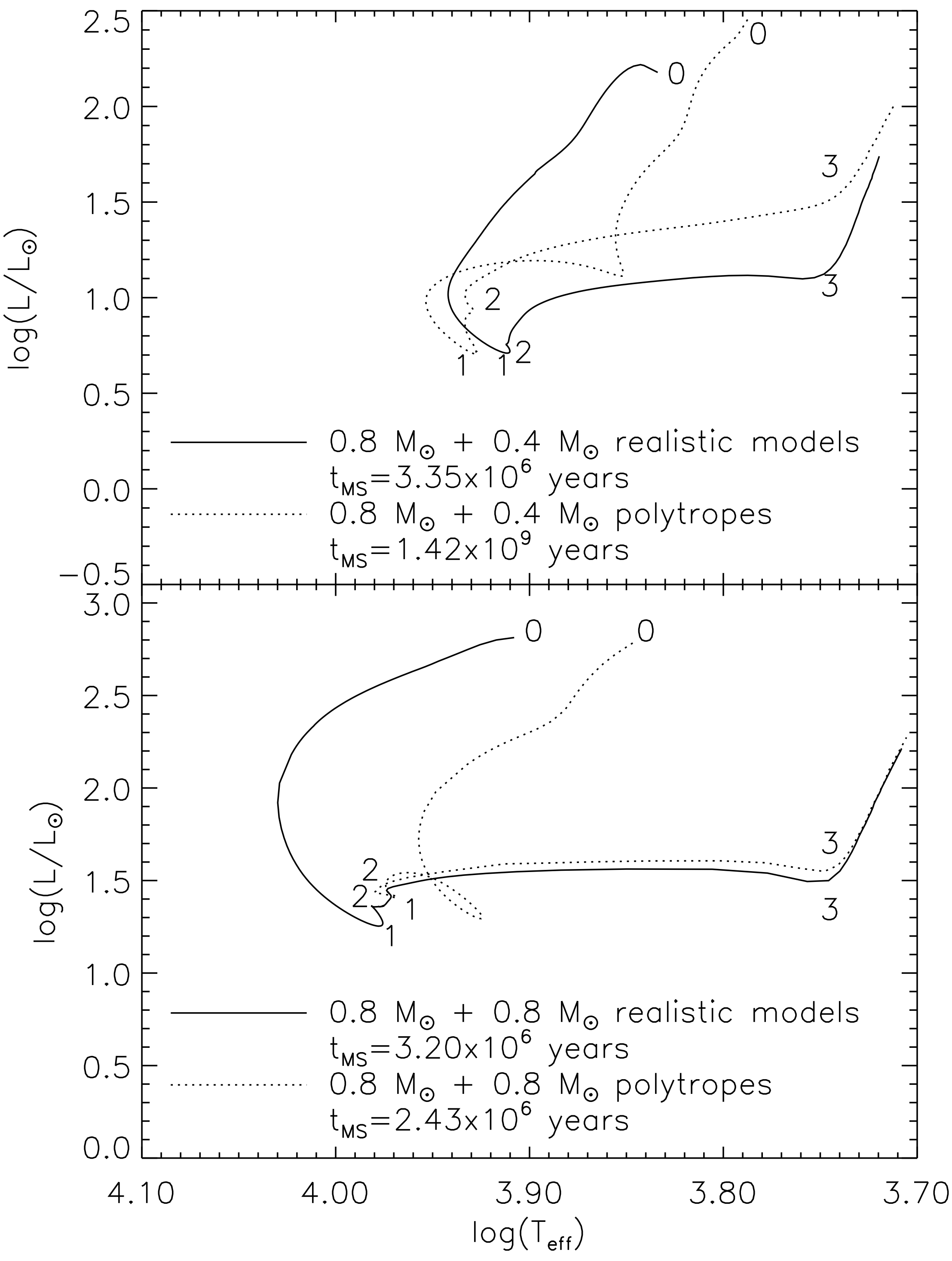


ln(P/ρ^5/3)
30
31
32
33
34
35
0.0
0.2
0.4
0.6
0.8
1.0
Mass Fraction
M=0.8⊙ polytrope (n=3)
M=0.8⊙ evolved to 15 Gyr
M=0.4⊙ polytrope (n=1.5)
M=0.4⊙ evolved to 15 Gyr



Case G: 0.8 $M_\odot$ + 0.4 $M_\odot$
$\log(L(r)/L_\odot)$
200
150
100
50
0
−50
0.0
0.2
0.4
0.6
0.8
1.0
Y
1.2
1.0
0.8
0.6
0.4
0.2
0.0
$\log(T(K))$
8
7
6
5
$\log(\rho(g/cm^3))$
6
4
2
0
−2
−4
−6
Mass Fraction
Mass Fraction


Case A: 0.8 M⊙ + 0.8 M⊙
log(L(r)/L⊙)
Y
log(T(K))
log(ρ(g/cm³))
Mass Fraction
Mass Fraction



0
0
3
3
2
1
1
2
log(L/L$_\odot$)
0.8 M$_\odot$ + 0.4 M$_\odot$ realistic models
t$_{MS}$=3.35x10^6 years
0.8 M$_\odot$ + 0.4 M$_\odot$ polytropes
t$_{MS}$=1.42x10^9 years
0
0
3
2
2
1
1
3
log(L/L$_\odot$)
0.8 M$_\odot$ + 0.8 M$_\odot$ realistic models
t$_{MS}$=3.20x10^6 years
0.8 M$_\odot$ + 0.8 M$_\odot$ polytropes
t$_{MS}$=2.43x10^6 years
log(T$_{eff}$)